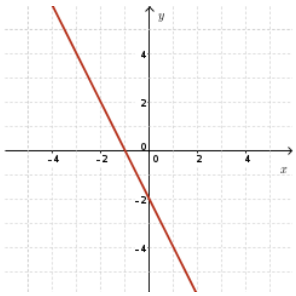
The Reciprocal of a Polynomial Function

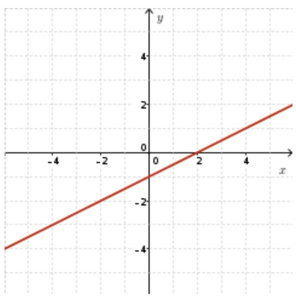
Exercises

1. Using the graph of the function $y = f(x)$ provided, sketch the graph of the reciprocal function $y = \frac{1}{f(x)}$.

a.



b.



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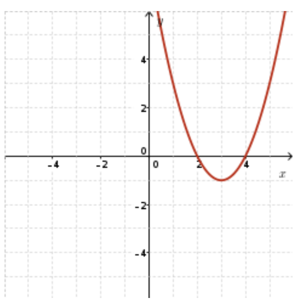

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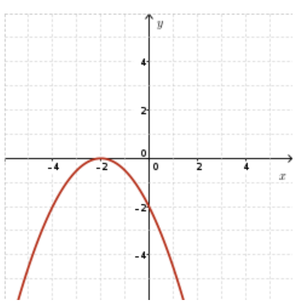
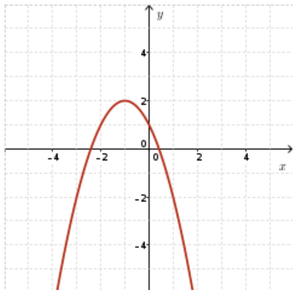


e. 

2. For each of the following functions, determine the domain, range, intercepts, positive/negative intervals, and intervals of increase/decrease. Use this information to sketch the graphs of the function, $y = f(x)$, and its reciprocal, $y = \frac{1}{f(x)}$.

   a. $f(x) = -2x + 6$
   b. $f(x) = x^2 + 3$
   c. $f(x) = -x^2 + 9$
   d. $f(x) = -x^2 - 2x + 3$

3. Sketch the graph of $y = f(x)$ and $y = \frac{1}{f(x)}$ on the same axes and state the domain and range for each function.

   a. $f(x) = -x - 2$
   b. $f(x) = (x + 5)^2$
   c. $f(x) = -(x - 3)^2 + 1$
   d. $f(x) = x^2 + x - 6$
   e. $f(x) = \frac{1}{2}x^2 - x - 4$

4. Sketch the graph of the function. Identify the domain and range, intercepts, positive and negative intervals, and intervals of increase and decrease.

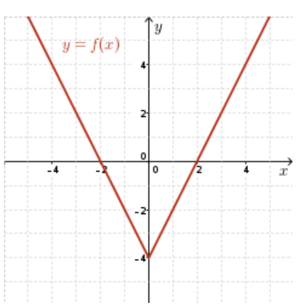
   a. $f(x) = \frac{1}{-2x + 5}$
   b. $f(x) = \frac{1}{x^2 + x}$
   c. $f(x) = \frac{1}{-x^2 - 5x - 3}$

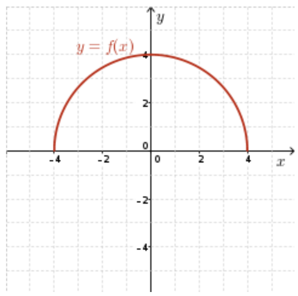
5. a. Using graphing technology (exercise investigation provided in this module), investigate the behaviour of the graphs of linear functions of the form $f(x) = x + n$ and their corresponding reciprocal functions of the form, $g(x) = \frac{1}{x + n}$, where $n \in \mathbb{R}$. How does changing the value of n affect the graphs of these functions? What points do the graphs of $f(x)$ and $g(x)$ have in common?

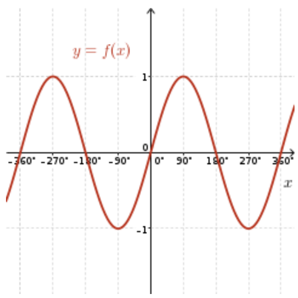
   b. Repeat part (a) using quadratic functions of the form $f(x) = x^2 + n$ and the corresponding reciprocal functions $g(x) = \frac{1}{x^2 + n}$.

   c. Repeat part (a) using quadratic functions of the form $f(x) = (x - n)^2$ and the corresponding reciprocal functions $g(x) = \frac{1}{(x - n)^2}$.

6. a. Given the graph of $y = f(x)$, sketch the graph of $y = \frac{1}{f(x)}$.

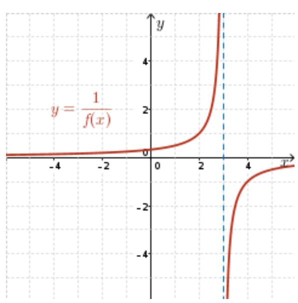
   i. 

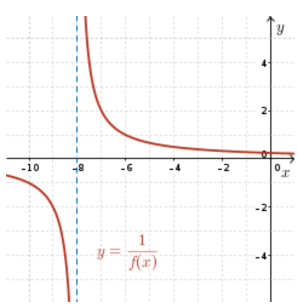
ii. 

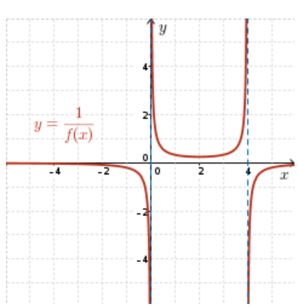
iii. 

b. For each function in part a), identify an equation for $y = f(x)$ and $y = \dfrac{1}{f(x)}$.

7. Given the graph of the reciprocal function $y = \dfrac{1}{f(x)}$, sketch the graph the function $y = f(x)$. Determine an equation for each function.

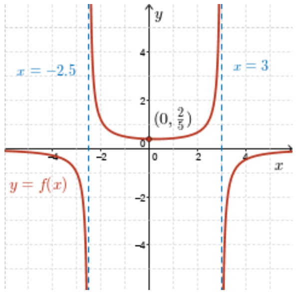
a. 

b. 

c. 

8. Determine an equation for each rational function of the form $f(x) = \dfrac{n}{ax^2 + bx + c}$, $n, a, b, c \in \mathbb{R}$. Justify your answer.

   a. A rational function, $y = f(x)$, with vertical asymptotes at $x = 2$ and $x = -\frac{1}{3}$, passing through $(3, -\frac{1}{2})$.

   b. 

9. Using your knowledge of transformations and your understanding of reciprocal functions, sketch each of the following functions.

   a. $f(x) = \dfrac{3}{(x-1)^2} + 2$

   b. $f(x) = \dfrac{-1}{x^2 - 4} - 3$

   c. $f(x) = \dfrac{-2}{\sqrt{x-3}}$

   d. $f(x) = \dfrac{1}{|x-4| - 1}$

10. A local community group is planning to hold a Valentine's Day dance at the community centre which holds a maximum of 500 people. Admission to the dance is free. The community group predicts it will cost about $\$3000$ from their budget to have this dance. Let p represent the number of people attending the dance and let C represent the average cost per person for the community centre.

    a. Determine an equation, for C in terms of p, to model this situation.

    b. State the domain of the function.

    c. Sketch a graph of the function.

    d. Is the graph increasing or decreasing? What does this mean in the context of the situation?

    e. How would the equation and graph of the function change if the community group receives $400 in donations to offset the cost of the dance?

11. Students in a local physics class are investigating Ohms law which relates the voltage, V, in an electrical circuit, measured in volts, to the electric current, I, measured in amperes, and the resistance, R, measured in ohms, by the equation $V = IR$. An electrical circuit is created by connecting a 9 volt battery to a variable resistor which allows the resistance to vary between 0 and 50 ohms.

    a. Determine a function to represent the current, I, passing through the circuit as a function of the resistance, r.

    b. What is an appropriate domain for this situation? Sketch a graph of the function.

    c. Students then insert a light bulb, with a resistance of 10 ohms, in series with the circuit to see how the variable resistor affects the brightness of the light bulb. How does the function determined in part (a) change? Recall that the total resistance in a series circuit is the sum of the resistance of all of the resistors in the circuit.

12. Two objects with masses m and M are a distance d apart, centre to centre. Newton's law of universal gravitation states that the force of gravitation between two objects, F, is inversely proportional to the square of the centre to centre distance d between them. That is $F = \dfrac{GmM}{d^2}$, where G is the universal gravitation constant.

    a. Sketch a graph of the function $F(d) = \dfrac{k}{d^2}$, for some constant k, $k > 0, k \in \mathbb{R}$. What is an appropriate domain for this situation?

    b. As the distance between the two objects increases, does the gravitational force increase or decrease?

    c. If the distance between the two objects doubles, by what factor does the gravitational force change?

    d. In order for the gravitational force to quadruple, how must the centre to centre distance change?

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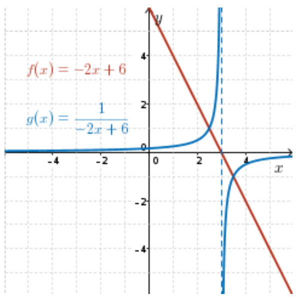
The Reciprocal of a Polynomial Function

Partial Solutions

1. There is no solution provided for this question.

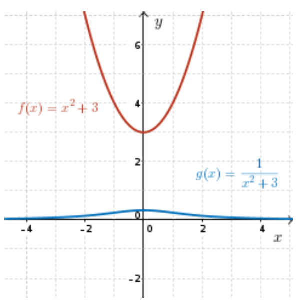
2. a. Domain: $\{x \mid x \in \mathbb{R}\}$, Range: $\{y \mid y \in \mathbb{R}\}$

Since the function $y = -2x + 6$ has an x-intercept at $x = 3$, the function $y = \frac{1}{-2x+6}$ will have a vertical asymptote at $x = 3$. The function $y = -2x + 6$ is a decreasing function for all x, $x \in \mathbb{R}$, so the function $y = \frac{1}{-2x+6}$ will be an increasing function over its domain, $x \neq 3, x \in \mathbb{R}$. The function $y = -2x + 6$ is positive for $x \in (-\infty, 3)$, so the function $y = \frac{1}{-2x+6}$ will also be positive in this region. Similarly, where $y = -2x + 6$ is negative, $x \in (3, \infty)$, the function $y = \frac{1}{-2x+6}$ will also be negative. As $x \to \pm\infty$, $\frac{1}{-2x+6} \to 0$, therefore $y = 0$ is the horizontal asymptote of $y = \frac{1}{-2x+6}$.



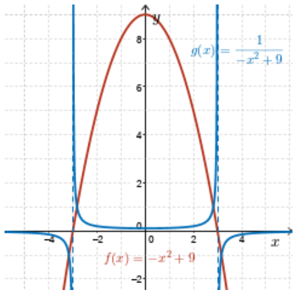
b. Domain: $\{x \mid x \in \mathbb{R}\}$, Range: $\{y \mid y \geq 3, y \in \mathbb{R}\}$

Since the function $y = x^2 + 3$ has no x-intercepts, the function $y = \frac{1}{x^2+3}$ will have no vertical asymptotes. Since the function $y = x^2 + 3$ is always positive, the function $y = \frac{1}{x^2+3}$ will also be positive over its domain, $x \in \mathbb{R}$. The function $y = x^2 + 3$ has a minimum at the point $(0, 3)$, thus the function $y = \frac{1}{x^2+3}$ will have a local maximum at the point $\left(0, \frac{1}{3}\right)$. In this situation $\left(0, \frac{1}{3}\right)$ is a maximum point on the curve. The function $y = x^2 + 3$ is decreasing for $x \in (-\infty, 0)$ and increasing for $x \in (0, \infty)$, so the function $y = \frac{1}{x^2+3}$ is increasing for $x \in (-\infty, 0)$ and decreasing for $x \in (0, \infty)$. As $x \to \pm\infty$, $\frac{1}{x^2+3} \to 0$. Therefore, $y = 0$ is the horizontal asymptote of $y = \frac{1}{x^2+3}$.



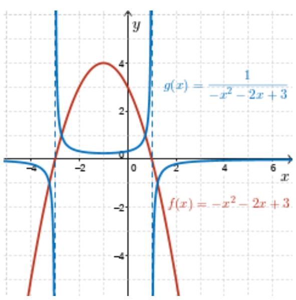
c. Domain: $\{x \mid x \in \mathbb{R}\}$, Range: $\{y \mid y \leq 9, y \in \mathbb{R}\}$

Since the function $y = -x^2 + 9$ has two x-intercepts, at $(-3, 0)$ and $(3, 0)$, the function $y = \frac{1}{-x^2+9}$ will have two vertical asymptotes, at $x = -3$ and $x = 3$. The function $y = -x^2 + 9$ is positive over $x \in (-3, 3)$, therefore the function $y = \frac{1}{-x^2+9}$ will also be positive in this interval. Similarly, where $y = -x^2 + 9$ is negative, $x \in (-\infty, -3) \cup (3, \infty)$, $y = \frac{1}{-x^2+9}$ will also be negative. The function $y = -x^2 + 9$ has a maximum at $(0, 9)$, so the function $y = \frac{1}{-x^2+9}$ will have a local minimum at $\left(0, \frac{1}{9}\right)$. The function $y = -x^2 + 9$ is increasing for $x \in (-\infty, 0)$ and decreasing for $x \in (0, \infty)$, so the function $y = \frac{1}{-x^2+9}$ is decreasing for $x \in (-\infty, -3) \cup (-3, 0)$ and increasing for $x \in (0, 3) \cup (3, \infty)$. As $x \to \pm\infty$, $\frac{1}{-x^2+9} \to 0$. Therefore, $y = 0$ is the horizontal asymptote of $y = \frac{1}{-x^2+9}$.



d. Domain: $\{x \mid x \in \mathbb{R}\}$, Range: $\{y \mid y \leq 4, y \in \mathbb{R}\}$

The equation of the function $y = -x^2 - 2x + 3$ can be expressed in factored form, $y = -(x+3)(x-1)$, to identify the x-intercepts. Since the function $y = -x^2 - 2x + 3$ has two x-intercepts, at $(-3, 0)$ and $(1, 0)$, the function $y = \dfrac{1}{-x^2 - 2x + 3}$ will have two vertical asymptotes, at $x = -3$ and $x = 1$. The vertex of the quadratic function is found on the axis of symmetry, $x = \dfrac{-3+1}{2} = -1$. The function $y = -x^2 - 2x + 3$ has a maximum at $(-1, 4)$, the function $y = \dfrac{1}{-x^2 - 2x + 3}$ will have a local minimum at $\left(-1, \frac{1}{4}\right)$. The function $y = -x^2 - 2x + 3$ is increasing for $x \in (-\infty, -1)$ and decreasing for $x \in (-1, \infty)$, so the function $y = \dfrac{1}{-x^2 - 2x + 3}$ is decreasing for $x \in (-\infty, -3) \cup (-3, -1)$ and increasing for $x \in (-1, 1) \cup (1, \infty)$. Since the function $y = -x^2 - 2x + 3$ is positive over $x \in (-3, 1)$, the function $y = \dfrac{1}{-x^2 - 2x + 3}$ will also be positive over this region. Similarly, where $y = -x^2 - 2x + 3$ is negative, $x \in (-\infty, -3) \cup (1, \infty)$, $y = \dfrac{1}{-x^2 - 2x + 3}$ will also be negative. As $x \to \pm\infty$, $\dfrac{1}{-x^2 - 2x + 3} \to 0$. Therefore, $y = 0$ is the horizontal asymptote of $y = \dfrac{1}{-x^2 - 2x + 3}$.



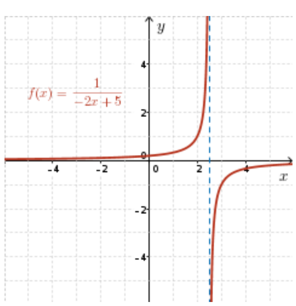
3. There is no solution provided for this question.

4. a. To graph the function $y = \dfrac{1}{-2x+5}$, we consider $y = -2x + 5$. Since the function $y = -2x + 5$ has an x-intercept at $\left(\frac{5}{2}, 0\right)$, the function $y = \dfrac{1}{-2x+5}$ will have a vertical asymptote at $x = \frac{5}{2}$. Thus the domain of $y = \dfrac{1}{-2x+5}$ is $\{x \mid x \neq \frac{5}{2}, x \in \mathbb{R}\}$.

Substituting $x = 0$ into $y = \dfrac{1}{-2x+5}$ determines that the y-intercept is $\left(0, \frac{1}{5}\right)$.

Since the function $y = -2x + 5$ is positive for $x \in \left(-\infty, \frac{5}{2}\right)$, the function $y = \dfrac{1}{-2x+5}$ will also be positive in this interval. Similarly, the function $y = -2x + 5$ is negative for $x \in \left(\frac{5}{2}, \infty\right)$, so the function $y = \dfrac{1}{-2x+5}$ will also be negative in this interval.

Since the function $y = -2x + 5$ is decreasing for all $x, x \in \mathbb{R}$, the function $y = \dfrac{1}{-2x+5}$ will be increasing over its domain. The function $y = \dfrac{1}{-2x+5}$ will have a horizontal asymptote of $y = 0$. The range of $y = \dfrac{1}{-2x+5}$ is $\{y \mid y \neq 0, y \in \mathbb{R}\}$.

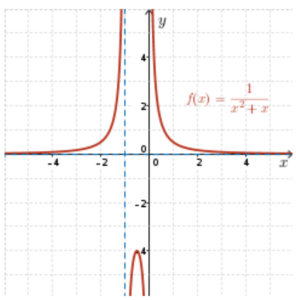


b. To graph the function $y = \dfrac{1}{x^2 + x}$, we consider $y = x^2 + x = x(x + 1)$. Since the function $y = x^2 + x$ has x-intercepts $(-1, 0)$ and $(0, 0)$, the function $y = \dfrac{1}{x^2 + x}$ will have vertical asymptotes at $x = -1$ and $x = 0$. Thus the domain of $y = \dfrac{1}{x^2 + x}$ is $\{x \mid x \neq -1, x \neq 0, x \in \mathbb{R}\}$.

$y = \dfrac{1}{x^2 + x}$ has no y-intercept since $x \neq 0$. The point $\left(-\frac{1}{2}, -\frac{1}{4}\right)$ is a minimum point on $y = x^2 + x$, so $\left(-\frac{1}{2}, -4\right)$ will be a local maximum for the function $y = \dfrac{1}{x^2 + x}$.

Since the function $y = x^2 + x$ is positive for $x \in (-\infty, -1) \cup (0, \infty)$, the function $y = \dfrac{1}{x^2 + x}$ will also be positive in this interval. By similar reasoning we can determine that $y = \dfrac{1}{x^2 + x}$ will be negative for $x \in (-1, 0)$.

Since the function $y = x^2 + x$ is decreasing for $x \in \left(-\infty, -\frac{1}{2}\right)$ the function $y = \dfrac{1}{x^2 + x}$ will be increasing for $x \in (-\infty, -1) \cup \left(-1, -\frac{1}{2}\right)$. By similar reasoning, the function $y = \dfrac{1}{x^2 + x}$ is decreasing for $x \in \left(-\frac{1}{2}, 0\right) \cup (0, \infty)$. The function $y = \dfrac{1}{x^2 + x}$ will have a horizontal asymptote of $y = 0$. The range of $y = \dfrac{1}{x^2 + x}$ is $\{y \mid y \leq -4 \text{ or } y > 0, y \in \mathbb{R}\}$.



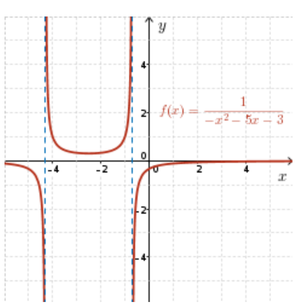
c. To graph the function $y = \dfrac{1}{-x^2 - 5x - 3}$, we consider $y = -x^2 - 5x - 3$. Since the function $y = -x^2 - 5x - 3$ has x-intercepts at $x = \dfrac{-5 \pm \sqrt{13}}{2}$, the function will have vertical asymptotes at $x = \dfrac{-5 \pm \sqrt{13}}{2}$. Thus, the domain of $y = \dfrac{1}{-x^2 - 5x - 3}$ is $\left\{x \mid x \neq \dfrac{-5 \pm \sqrt{13}}{2}, x \in \mathbb{R}\right\}$.

Substituting $x = 0$ into $y = \dfrac{1}{-x^2 - 5x - 3}$ determines that the y-intercept is $\left(0, -\frac{1}{3}\right)$. The point $\left(-\frac{5}{2}, \frac{13}{4}\right)$ is a maximum point on $y = -x^2 - 5x - 3$, so $\left(-\frac{5}{2}, \frac{4}{13}\right)$ will be a local minimum for the function $y = \dfrac{1}{-x^2 - 5x - 3}$.

Since $y = -x^2 - 5x - 3$ is positive for $x \in \left(\frac{-5-\sqrt{13}}{2}, \frac{-5+\sqrt{13}}{2}\right)$, the function $y = \dfrac{1}{-x^2 - 5x - 3}$ will also be positive in this interval. By similar reasoning, we can determine that $y = \dfrac{1}{-x^2 - 5x - 3}$ will be negative for $x \in \left(-\infty, \frac{-5-\sqrt{13}}{2}\right) \cup \left(\frac{-5+\sqrt{13}}{2}, \infty\right)$.

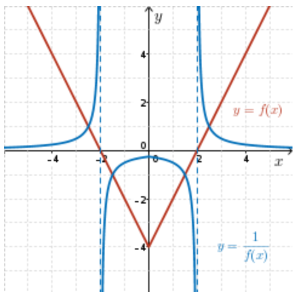
Since the function $y = -x^2 - 5x - 3$ is decreasing for $x \in \left(-\frac{5}{2}, \infty\right)$ we can determine that the function $y = \dfrac{1}{-x^2 - 5x - 3}$ will be increasing for $x \in \left(-\frac{5}{2}, \frac{-5+\sqrt{13}}{2}\right) \cup \left(\frac{-5+\sqrt{13}}{2}, \infty\right)$. By similar reasoning we can determine that the function $y = \dfrac{1}{-x^2 - 5x - 3}$ will be decreasing for $x \in \left(-\infty, \frac{-5-\sqrt{13}}{2}\right) \cup \left(\frac{-5-\sqrt{13}}{2}, -\frac{5}{2}\right)$.

The function $y = \dfrac{1}{-x^2 - 5x - 3}$ will have a horizontal asymptote of $y = 0$. The range of $y = \dfrac{1}{-x^2 - 5x - 3}$ is $\{y \mid y < 0 \text{ or } y \geq \frac{4}{13}, y \in \mathbb{R}\}$.

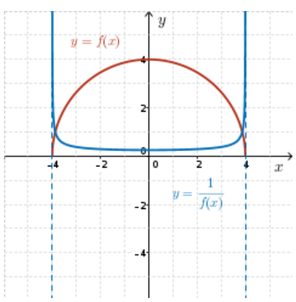


5. There is no solution provided for this question.

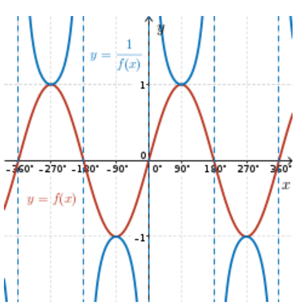
6. a. i. The graph of $y = f(x)$ has x-intercepts at $(-2, 0)$ and $(2, 0)$, the graph of $y = \frac{1}{f(x)}$ will have vertical asymptotes at $x = -2$ and $x = 2$. Since the graph of $y = f(x)$ has a minimum at $(0, -4)$, the graph of $y = \frac{1}{f(x)}$ will have a local maximum at $\left(0, -\frac{1}{4}\right)$. The graph of $y = \frac{1}{f(x)}$ will be positive for $x \in (-\infty, -2) \cup (2, \infty)$ and negative for $x \in (-2, 2)$ as these are the regions that correspond to where $y = f(x)$ is positive and negative, respectively. (Recall that the graph of $y = \frac{1}{f(x)}$ will be positive where the graph of $y = f(x)$ is positive, and negative where $y = f(x)$ is negative). As $x \to \pm\infty, f(x) \to \infty$, therefore $\frac{1}{f(x)} \to 0$. Thus, $y = \frac{1}{f(x)}$ has a horizontal asymptote of $y = 0$.



ii. Since the graph $y = f(x)$ has x-intercepts at $(-4, 0)$ and $(4, 0)$, the graph of $y = \frac{1}{f(x)}$ will have vertical asymptotes at $x = -4$ and $x = 4$. The graph of $y = f(x)$ has a maximum at $(0, 4)$, so the graph of $y = \frac{1}{f(x)}$ will have a local minimum at $(0, \frac{1}{4})$. The graph of $y = \frac{1}{f(x)}$ will only be defined for $x \in (-4, 4)$ as the domain of $y = f(x)$ is $x \in [-4, 4]$. As $x \to \pm 4, f(x) \to 0$ from above. Therefore, $\frac{1}{f(x)} \to \infty$.



iii. $y = f(x)$ is an oscillating function with a period of 360°. It has x-intercepts at intervals of $180^\circ n, n \in \mathbb{Z}$, thus the graph of $y = \frac{1}{f(x)}$ will have vertical asymptotes at intervals of $180^\circ n, n \in \mathbb{Z}$. Since the graph of $y = f(x)$ has maximum points at $(90^\circ + 360^\circ k, 1)$ where $k \in \mathbb{Z}$, the graph of $y = \frac{1}{f(x)}$ will have local minimums at $(90^\circ + 360^\circ k, 1)$ where $k \in \mathbb{Z}$. Similarly, since the graph of $y = f(x)$ has minimums at $(270^\circ + 360^\circ k, -1)$ where $k \in \mathbb{Z}$, the graph of $y = \frac{1}{f(x)}$ will have local maximums at $(270^\circ + 360^\circ k, -1)$ where $k \in \mathbb{Z}$. As the graph of $y = f(x)$ approaches the x-axis from above, the graph of $y = \frac{1}{f(x)}$ will approach positive infinity and as the graph of $y = f(x)$ approaches the x-axis from below, the graph of $y = \frac{1}{f(x)}$ will approach negative infinity.



b. i. The function $y = f(x)$ is the absolute value function, $y = |x|$, stretched vertically about the x-axis by a factor of 2 and translated down

4 units. (Notice from the corner point $(0, -4)$ the graph has a rise of 2 and a run of 1). This suggests that there has been a vertical stretch by a factor of 2 or a horizontal stretch by a factor of $\frac{1}{2}$ applied to the base graph of $y = |x|$.) Therefore, possible equations for the function $y = f(x)$ are $y = 2|x| - 4$ or $y = |2x| - 4$. Thus equations for $y = \frac{1}{f(x)}$ are $y = \frac{1}{2|x| - 4}$ or $y = \frac{1}{|2x| - 4}$, respectively.

ii. The graph of $y = f(x)$ is the positive half of a circle with a radius of 4. Recall the general equation of a circle with radius r is $x^2 + y^2 = r^2$. Thus, the equation for a circle with radius 4 is $x^2 + y^2 = 16$.

$$x^2 + y^2 = 16$$
$$y^2 = 16 - x^2$$
$$y = \pm\sqrt{16 - x^2}$$

The equation of the half circle where $y \geq 0$ is then $y = \sqrt{16 - x^2}$. Thus an equation for $y = f(x)$ is $y = \sqrt{16 - x^2}$ and an equation for $y = \frac{1}{f(x)}$ is $y = \frac{1}{\sqrt{16 - x^2}}$.

iii. $y = f(x)$ is an oscillating function with a period of 360°. The graph oscillates between $y = 1$ and $y = -1$. Based on the position of the zeros and minimum/maximum points, a possible equation for $y = f(x)$ is $y = \sin(x)$. Thus an equation for $y = \frac{1}{f(x)}$ is $y = \frac{1}{\sin(x)}$ or $y = \csc(x)$.

7. There is no solution provided for this question.

8. a. Since the vertical asymptotes are $x = 2$ and $x = -\frac{1}{3}$, then $x = 2$ and $x = -\frac{1}{3}$ are zeros of the quadratic polynomial in the denominator. Therefore, $(x - 2)$ and $(3x + 1)$ are factors of the denominator. So, $f(x) = \frac{n}{a(x-2)(3x+1)}$.

Substituting in the point, $\left(3, -\frac{1}{2}\right)$, which must satisfy the equation, we have

$$-\frac{1}{2} = \frac{n}{a(3-2)(3(3)+1)}$$
$$-\frac{1}{2} = \frac{n}{a(1)(10)}$$
$$-\frac{1}{2} = \frac{n}{10a}$$
$$\therefore \frac{n}{a} = -5$$

Setting $a = 1, n = -5, \therefore f(x) = \frac{-5}{3x^2 - 5x - 2}$ is a possible equation.

b. Since the vertical asymptotes are $x = -2.5 = -\frac{5}{2}$ and $x = 3$, then $x = -\frac{5}{2}$ and $x = 3$ are zeros of the quadratic polynomial in the denominator. Therefore, $(2x + 5)$ and $(x - 3)$ are factors of the denominator. So, $f(x) = \frac{n}{a(2x+5)(x-3)}$.

Substituting in the y-intercept, we have $\left(0, \frac{2}{5}\right)$, which must satisfy the equation, we have

$$\frac{2}{5} = \frac{n}{a(2(0)+5)(0-3)}$$
$$\frac{2}{5} = \frac{n}{a(5)(-3)}$$
$$\frac{2}{5} = \frac{n}{-15a}$$
$$\therefore \frac{n}{a} = -6$$

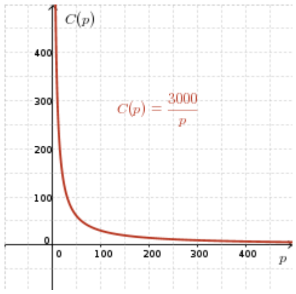
Setting $a = 1$ and $n = -6$, we have that $f(x) = \frac{-6}{2x^2 - x - 15}$ is a possible equation.

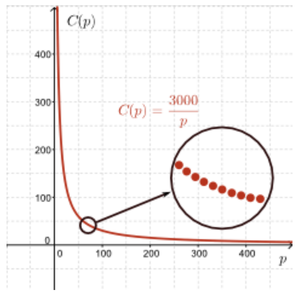
9. There is no solution provided for this question.

10. a. To determine the average cost per person, we divide the total cost of the dance by the number of people that attend the dance. Thus, the equation, for C in terms of p, to model this situation is $C(p) = \frac{3000}{p}$.

b. To determine the domain, we must consider the number of people that are able to attend the dance. Since p represents the number of people attending the dance and $C(p)$ is not defined when p is 0, p is an integer greater than 0. The maximum capacity of the community centre is 500 people. Thus the domain of the function is $0 < p \leq 500, p \in \mathbb{Z}$.

c. The graph of $C(p) = \frac{3000}{p}, 0 < p \leq 500, p \in \mathbb{Z}$ will be the graph of $C(p) = \frac{1}{p}, 0 < p \leq 500, p \in \mathbb{Z}$ with a vertical stretch of 3000.



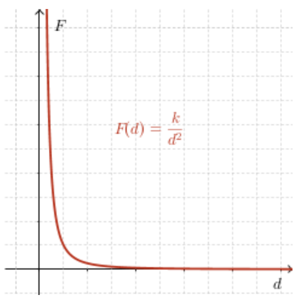


d. By looking at the graph, we can see that the function is decreasing over its domain. This means that, as the number of people attending the dance increases, the average cost per person decreases.

e. If the community group raises $\$400$ selling refreshments then the overall cost to the community group for the dance will decrease to $\$3000 - \$400 = \$2600$. Thus, the equation will be $C(p) = \frac{2600}{p}$. The vertical stretch applied to $C(p) = \frac{1}{p}$ is reduced from a factor of 3000 to a factor of 2600. Thus, the graph will appear closer to the x- and y-axes.

11. There is no solution provided for this question.

12. a. First note that GmM can be set to some constant $k, k > 0, k \in \mathbb{R}$ since m and M, the masses of the two objects are constant and G is the universal gravitational constant. The graph of $F(d) = \frac{k}{d^2}$ is the graph of $F(d) = \frac{1}{d^2}$ with a vertical stretch by a factor of some constant k (no reflection since $k > 0$). Since d represents the distance between two objects then d must be a positive real value. The domain for this situation is $\{d \mid d > 0, d \in \mathbb{R}\}$. Thus, the graph of the function does not include the branch of the curve where the value of d is negative.



b. Since the graph of $F(d) = \frac{k}{d^2}$ is decreasing, as the distance between the two objects increases, the gravitational force will decrease.

c. If the distance between two objects doubles, then the distance changes from d to $2d$. Then,

$$\begin{aligned} F(2d) &= \frac{GmM}{(2d)^2} \\ &= \frac{GmM}{4d^2} \\ &= \frac{1}{4}\left(\frac{GmM}{d^2}\right) \\ &= \frac{1}{4}F(d) \end{aligned}$$

Thus, the gravitational force will be $\frac{1}{4}$ of the original gravitational force.

d. If the gravitational force quadruples, then the force is multiplied by 4. To determine how d must change, manipulate the equation

$$\begin{aligned} 4F(d) &= 4\left(\frac{GmM}{d^2}\right) \\ &= \frac{GmM}{\frac{1}{4}d^2} \\ &= \frac{GmM}{\left(\frac{1}{2}d\right)^2} \end{aligned}$$

Therefore, the distance must be $\frac{1}{2}$ the original distance.

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